REPORT DOCUMENTATION PAGE	Form Approved OMB NO. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimates or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE July 1997	3. REPORT TYPE AND DATES COVERED Technical – July'97

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
Local eigenfunctions based suboptimal wavelet packet representation of contaminated chaotic signals	DAAH04-96-1-0082

6. AUTHOR(S)

Satish T.S. Bukkapatnam, Slundar R.T. Kumara and A. Lakhtakia

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Center for Multivariate Analysis 417 Thomas Building Department of Statistics Penn State University University Park, PA 16802	

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING / MONITORING AGENCY REPORT NUMBER
U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	ARO 35518.20-MA

11. SUPPLEMENTARY NOTES

The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12a. DISTRIBUTION / AVAILABILITY STATEMENT	12 b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words)

We report a suboptimal wavelet packet (WP) representation of signals emanating from a chaotic attractor contaminated by low levels of noise. Our method - geared towards choosing a suboptimal scaling function to parsimoniously represent the signal - involves extracting local eigenfunctions using artificial ensembles generated from a *pseudo-probability space*, and using the extracted local eigenfunctions to develop a suboptimal scaling function.

The application of our novel representation method to actual acoustic emission (AE) signals from the turning process reveals the superiority of these methods over the existing signal representations.

DTIC QUALITY INSPECTED 4

14. SUBJECT TERMS Karhunen-Loeve representation, Wavelet representation, Biorthogonal wavelet representation, Chaotic signals, Pseudo probability space, Wavelet packet, Local eigenfunction representation, Attractor, Poincare section, Decorrelation length		15. NUMBER IF PAGES 23
		16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

LOCAL EIGENFUNCTIONS BASED SUBOPTIMAL WAVELET

PACKET REPRESENTATION OF CONTAMINATED CHAOTIC SIGNALS

S.T.S. Bukkapatnam, S. Kumara and A. Lakhtakia

Technical Report 97-14

July 1997

Center for Multivariate Analysis
417 Thomas Building
Penn State University
University Park, PA 16802

19970819 024

Local eigenfunctions based suboptimal wavelet packet representation of contaminated chaotic signals

Satish T. S. Bukkapatnam   Soundar R. T. Kumara

Department of Industrial and Manufacturing Engineering

Akhlesh Lakhtakia

Department of Engineering Science and Mechanics

The Pennsylvania State University

University Park PA 16802

U. S. A.

Abstract

We report a suboptimal wavelet packet (WP) representation of signals emanating from a chaotic attractor contaminated by low levels of noise. Our method—geared towards choosing a suboptimal scaling function to parsimoniously represent the signal—involves extracting local eigenfunctions using artificial ensembles generated from a *pseudo-probability space*, and using the extracted local eigenfunctions to develop a suboptimal scaling function.

The application of our novel representation method to actual acoustic emission (AE) signals from the turning process reveals the superiority of these methods over the existing signal representations.

1   Introduction

Signal representation is an essential step in many engineering signal processing applications, such as pattern recognition and state estimation. The key task of signal representation is to find a basis to *parsimoniously* represent a given signal. In addition, the basis should (i) capture essential signal features such as discontinuities, (ii) match the smoothness of the signal, (iii) accomodate rapid fluctuations in the signal, (iv) cater to the stochasticity and the distribution of the signal, and (v) be usable on-line.

If the signal belongs to a separable space such as the Hilbert space, we can compactly represent the signal by a countable basis, i.e., there are at the most countable infinity of elements (here, the basis functions) in the basis. For example, suppose the signal emanates from a second order stochastic process, i.e., a process whose autocorrelation function remains finite and integrable over a specified interval. The space of second order processes $L_2(\mathbf{R}, \mathcal{B}, \mu)$—where $\mathcal{B}$ is the Borel-algebra constructed on the real line $\mathbf{R}$ and μ is the underlying probability measure—is complete and separable; therefore we may employ Karhunen–Loeve (KL) representation to obtain a basis.

The KL representation of a second order stochastic process $y(t)$ is given by

$$y(t) = \sum_{j \in \mathbf{Z}} a_j \alpha_j(t),$$ (1)

where $\mathbf{Z}$ is the set of integers, and $\alpha_j(t)$ are the independent solutions of

$$\int_{\mathbf{R}} K(t, \tau) \alpha_j(\tau) d\tau = \lambda_j \alpha_j(t)$$ (2)

2

(i.e., $\alpha_j(t)$ are the eigenfunctions of the covariance function $K(t,\tau)$). Since $K(t,\tau)$ is self-adjoint [9], the KL representation consists of expressing a given signal as a linear combination of a few orthonormal modes. Since the basis is orthonormal and consists of the eigenfunctions of the stochastic process underlying a measured signal, the KL representation is optimal in the mean square sense.

But, computing eigenfunctions of a generic second order process is extremely tedious. In addition, from an implementation standpoint, the KL representation needs a few ensembles of a stochastic sequence to estimate $K(t,\tau)$. In the absence of ensembles, i.e., when only one realization is available, we somehow need to generate some artificial ensembles in order to develop a KL-like representation.

Alternatively, Fourier and wavelet representations are optimal under certain conditions and are commonly used in practice [3]. But they do not use eigenfunction basis. Of all the non-eigenfunction bases, wavelet bases are currently the most attractive, because of their ability to satisfy the five desirable properties mentioned earlier. By proper selection of wavelet basis, one may develop optimal representations of many classes of signals [8, 11].

Wavelet representation is algorithmically simple for representing transient signals in L_2-space. Many researchers have focussed on finding optimal wavelet basis. Coifman and Wickerhauser [12] in their best basis formulation proposed that, given an overcomplete set of basis functions belonging to a wavelet packet (WP) library, we may choose their optimal combination through a branch-and-bound method of entropy minimization. However, the WP library and hence the basis functions themselves are not chosen optimally with respect to the considered class of signals.

3

For the past one decade, several researchers have attempted to determine the optimal wavelet basis for different signal classes. Odegard et al. [8] developed optimal finite support wavelet basis to represent band-limited signals by minimizing the induced norm of the resolution error operator at every scale with respect to the scaling function of that scale. Their "robust" basis minimizes the worst-case error for all signals limited to a particular frequency band.

Unser [11] showed that, when the signal is a realization of a stationary process, the optimal scaling function is the impulse response of an ideal bandpass filter whose shape is determined by the signal energy. His analysis showed that the determination of an optimal scaling function for *biorthogonal wavelet representation* of a generic signal in L_2 space is mathematically intractable.

Strintzis [10] developed the necessary conditions for a globally optimal scaling function of a band-limited multidimensional signal, represented using biorthogonal wavelets. He found that computing a globally optimal scaling function for a generic multidimensional signal is mathematically intractable and, in many cases, not realizable.

Computationally simple methods are commonly used to determine suboptimal scaling functions [11, 10]. Either the structure of the synthesis filter (i.e., the filter to perform inverse wavelet transform) is assumed *a priori*, and the analysis filter (filter for wavelet transform) is constructed therefrom; or a Butterworth-filter-type approximation of the ideal unrealizable analysis-synthesis pair is performed.

However, most of the earlier works assume the signal to emanate from a stationary stochastic process with a known variance. None of the previous works explicitly addressed the issue of the optimal representation of chaotic signals. Realistically, con-

4

inated chaotic signals are more common than stationary signals. Hence, research finding the optimal representation of contaminated chaotic signals seems to be in der.

In this paper, we develop a suboptimal scaling function using the local eigenfunctions, extracted as described in the following two sections, to represent signals emanating from a chaotic process contaminated with low levels of noise. In other words, we develop a suboptimal wavelet packet (WP) representation of a signal which is a single realization of a chaotic process contaminated with low intensity noise.

Our methodology consists of (i) generating artifical ensembles from a *pseudo-probability space*, constructed from a measured signal, as explained in Section 2; (ii) extracting local eigenfunctions from the pseudo-probabability space, as described in Section 3 (these local eigenfunctions may be used to develop a computationally expensive KL-like representation scheme—called the local eigenfunction representation—from the obtained ensembles); and (iii) using the extracted local eigenfunctions for suboptimal WP representation, as described in Section 4.

2 Constructing pseudo-probability space of a chaotic attractor

Since the considered signal corresponds to a single trajectory of a contaminated chaotic process, the KL representation cannot be directly used. However, by appropriately constructing a *pseudo-probability space* and an associated proabability measure from the TSD, we can extract artificial ensembles to develop a KL-like representation. The

constructing of such a space and the associated probability measure for a dynamical system is studied in *ergodic theory* [4], which broadly deals with developing *invariant measures* for a dynamic system. Before we proceed further towards constructing the pseudo-probability space, let us review certain relevant aspects of dynamic systems and reinforce the concept of invariant measures.

2.1 Overview of ergodic theory and our approach

Suppose a dynamic system is modeled in terms of autonomous [7] nonlinear stochastic differential/difference equations with a state vector $\underline{x}(t) \in \mathbf{M}$ (the state space) and a discrete output $y(n) \equiv y(t_n)$ as follows:

$$d\underline{x} \;=\; \underline{F}(\underline{x})\,dt + \underline{\underline{g}}(\underline{x})\,d\underline{\beta}\,, \tag{3}$$

$$y(n) \;=\; h(\underline{x}(n)) + v(n) \;;\; n = 0, 1, 2, \ldots N\,. \tag{4}$$

Here, (3) represents the *process sytem* and (4) represents the *measurement setup*. Furthermore, $\underline{F}(\cdot)$ is a nonlinear stochastic vector field, $\underline{\underline{g}}(\cdot)$ and $h(\cdot)$ are continuous transformations, $\underline{\beta}$ is a vector Wiener process [9] that accounts for dynamic noise, $\underline{x}(n) \equiv \underline{x}(t_n)$, and $v(n)$ is a Gaussian white noise sequence [5] that accounts for additive measurement noise. As this is an autonomous system, $\underline{F}(\underline{x})$ and $\underline{\underline{g}}(\underline{x})$ are not explicit functions of time t. Equation (3) is an Itó equation [5] because the gain matrix $\underline{\underline{g}}(\underline{x})$ is a stochastic process.

When there is no dynamic noise, i.e., $\underline{\underline{g}}(\underline{x})\,d\underline{\beta} \equiv 0$, the solution to (3) results in a

6

trajectory

$$\underline{x}(t) = \underline{f}(\underline{x}(0), t),$$

(5)

where $\underline{f} : \mathbf{M} \mapsto \mathbf{M}$ represents the *flow* that determines the evolution of $\underline{x}(t)$ from a specific initial condition $\underline{x}(0)$. If the sytem is *dissipative*, i.e., the volume elements[1] in the state space contract as the system evolves,[2] then the trajectory generally tends asymptotically to certain compact subsets. If $\mathbf{S} \subset \mathbf{M}$ is a compact set and $\mathbf{U}$ is the largest open set that asymptotically contracts to $\mathbf{S} \subset \mathbf{U}$, then $\mathbf{S}$ is called an *attracting set* and $\mathbf{U}$ is called the *basin of attraction* of $\mathbf{S}$. An attracting set may be reduced into certain distinct portions, some of which may not be attracting.

All disjoint subsets $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \ldots$ of $\mathbf{S}$ that are attracting are called *attractors*, while $\mathbf{S} - (\bigcup_j \mathbf{A}_j)$ is non-attracting. An attractor $\mathbf{A} \subset \mathbf{M}$ is associated with the following four properties [7, 13]:

1. **Invariance:** If $\underline{x}(0) \in \mathbf{A}$, then $\underline{f}(\underline{x}(0), t) \in \mathbf{A}$ $\forall t > 0$.

2. **Attractivity:** For some $\mho \subset \mathbf{M}$ defined in the neighborhood of $\mathbf{A}$, $\underline{f}[\mho, t] \subset \mho$ and $\lim_{t \to \infty} \underline{f}[\mho, t] = \mathbf{A}$.

3. **Recurrence:** For every $\epsilon > 0$ and almost every $\underline{x}(0) \in \mathbf{A}$, $\exists\, t > 0$ such that $\|\underline{x}(0) - \underline{x}(t)\| < \epsilon$; in effect, the trajectories within an attractor remain bounded.

4. **Indecomposability:** An attractor is disjoint from all other attractors and

---

[1] Volume elements refer to the small finite chunks of state space.
[2] The concept of dissipative systems emerges from an Eulerian perspective rather than from a Lagrangian one.

7

cannot be split into nontrivial partitions satisfying the three aforementioned properties.

In dissipative systems, even though the overall size of a volume element decreases, there can be some directions along which the linear dimension of the volume element actually expands. However, as the attractors are bounded, the flow then exhibits a horseshoe-type pattern [13] so that trajectories starting from near-by points within an attractor A may get separated exponentially as the system evolves. This condition is known as the *sensitive dependence on initial condition*, and the attractor A is then called a *strange attractor*. A flow $\underline{f}(\cdot,\cdot)$ for a particular initial condition, is said to be chaotic if the trajectories in an attractor exhibit:

1. sensitive dependence on initial conditions but are bounded,

2. irregular and aperiodic behavior, and

3. continuous broad band spectrum.

Under the absence of dynamic as well as measurement noise, we can define an invariant probability measure on the attractor of a chaotic process as follows:

Definition 1 *A measure μ_A—which specifically refers to the probability measure in this context—defined on A with respect to $\underline{f}(\cdot,\cdot)$ is said to be invariant iff $\mu_A(A) = \mu_A(\underline{f}[A,t])$ holds for an at least countably many values of t.*

If the trajectory lies in the attractor, an invariant distribution can be naturally obtained because the attractor is a compact set and is invariant under the flow $\underline{f}(\cdot,\cdot)$. The existence of an invariant measure for an attractor is evident from the following theorem [6]:

8

Theorem 1 *Every continuous dynamic system defined on a compact space admits at least one invariant probability measure.*

Let us denote the probability space constructed from an admissible probability measure μ_A as $(\mathbf{A}, \mathcal{B}_A, \mu_A)$. The probability measure may be extracted using the *Poincaré recurrence theorem* stated as follows [6].

Theorem 2 *Let $\underline{\rho}: \mathbf{A} \rightarrow \mathbf{A}$ be a measurable transformation on a probability space $(\mathbf{A}, \mathcal{B}_A, \mu_A)$. Let $\Lambda \in \mathcal{B}_A$ have $\mu_A(\Lambda) > 0$. Then for almost all $\underline{x} \in \Lambda$, the orbit $\{\underline{\rho}^k(\underline{x})\}_{k \geq 0}$ returns to Λ infinitely often.*

Specifically, if one chooses Λ to be a Poincaré section of $\mathbf{A}$ passing through $\underline{x}_0$, by definition of a Poincaré section Λ is compact and invariant with respect to the Poincaré map

$$\underline{\rho}^k(\underline{x}) = \underline{f}(\underline{x}(0), t_k), \; t_k \geq 0, \tag{6}$$

where t_k is the time at which the trajectory enters the Poincaré section Λ. Therefore, by Theorem 1, Λ admits a measure μ_Λ invariant with respect to $\underline{\rho}(\cdot)$. Thus, the elements of Λ are separate independent samples of the probability space $(\Lambda, \mathcal{B}_\Lambda, \mu_\Lambda)$, where $\mathcal{B}_\Lambda$ is the Borel algebra constructed on Λ.

If Λ is assumed to lie in a small neighborhood of $\underline{x}(0) \in \mathbf{A}$ on the trajectory, i.e., $\Lambda \equiv cl[\mathbf{B}_\epsilon(\underline{x}(0))]$, a closed ball of size ϵ constructed about $\underline{x}(0)$, then the local evolutions from the elements of Λ are independent realizations and hence separate events.

Thus, local evolutions from neighborhood points of a single trajectory through $\underline{x}(0)$ may be treated as independent random trajectories emerging from the neighborhood of

9

$\underline{x}(0)$. In addition, for a chaotic process, the independent random trajectories emerging from the neighborhood of a certain initial point $\underline{x}(0) \in \mathbf{A}$ remain locally close together. Therefore, $(\Lambda, \mathcal{B}_\Lambda, \mu_\Lambda)$ is a valid probability space with the trajectory-segments emerging from Λ as realizations of a stochastic process defined thereon.

However, when the dynamic system is contaminated, the probability space constructed from a trajectory thereof is not valid under all conditions because the presence of noise may destroy the structure of the attractor $\mathbf{A}$. Hence, we call the constructed probability space a *pseudo-probability space*. Thus, from the constructed pseudo-probability space, the separate realizations consists of local evolutions from the neighborhood of $\underline{x}(0)$, all belonging to a single trajectory.

2.2   Pseudo-probability space construction procedure

In the foregoing, we assumed knowledge of $\underline{x}(t) \in \mathbf{A} \subset \mathbf{M}$. But when only a scalar TSD $y(n)$ corresponding to the process output is available, we need to extract samples using the concept of pseudo-probability space, which in turn requires the topological notion of neighborhood. For every $y(n)$, there exists a diffeomorphism mapping $y(n)$ into the observable portion of $\underline{x}(t)$. Alternatively, if we reconstruct the attractor of the original dynamics from the TSD using lag–coordinates [2], there is a diffeomorphism connecting the reconstructed state vector $\underline{\zeta}(\cdot)$ and observable subspace of the actual state vector $\underline{x}(\cdot)$. The state vector $\underline{\zeta}(\cdot)$ of the lag–reconstructed dynamics lies in a vector space of dimension d_E, the embedding dimension. We would like to stress that the procedure for determining an estimate of d_E and obtaining $\underline{\zeta}(n)$ is nontrivial. For the details thereof, the reader is referred to [1].
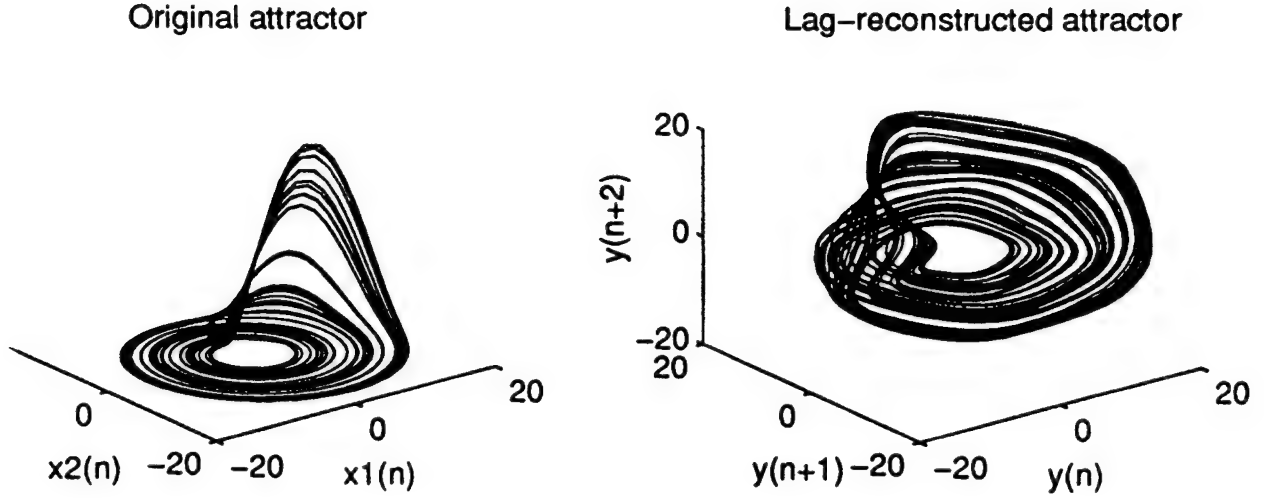
Original attractor

Lag–reconstructed attractor

igure 1: Rossler attractor obtained from: (a) actual solution of the differential equation,) lag coordinates.

We illustrate the construction of the pseudo-proabability space through the following example. The original attractor and the lag–reconstructed attractor of the Rossler attractor corresponding to the three first order differential equations

$$
\underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -(x_2 + x_3) \\ x_1 + 0.15x_2 \\ 0.20 + x_3(x_1 - 10.0) \end{bmatrix},
\tag{7}
$$

with $y(n) = x_1(t_n)$, $n = 1, 2, \ldots N$ are shown in Figure 6. In the lag–reconstructed vector space, at a particular point $\underline{\zeta}(n)$ on the attractor, the set of points $\{\underline{\zeta}(n \pm \ell), \ell = 0, 1, 2, \ldots\}$ resulting from the evolution (both forward as well as backward) of $\underline{\zeta}(n)$ constitutes a *strand*. If the TSD is chaotic, two nearby points located in different strands of the same trajectory (read TSD), locally stay close together before exponentially diverging-off.

Thus, if we can identify a set of N_B neighbors $\{\underline{\zeta}^r(n), 1 \le r \le N_B\}$ about a

11

specified point $\underline{\zeta}(n)$, belonging to different strands of the reconstructed trajectory, the evolutions of these neighbors approximate the ensembles required to compute the eigenfunctions. The length of the ensembles may be set equal to the length of the strand, which usually is the *decorrelation length* L of the TSD [1]. From the artificial ensembles generated thus, we may obtain the eigenfunctions using a similar procedure as in KL representation.

3 Extracting local eigenfunctions

From the artifical ensembles generated as described in the previous section, we can obtain locally optimal basis functions to represent L-length signal ensembles emerging from $\Lambda \equiv \mathbf{B}(\underline{x}(0))$, an open-ball about $\underline{x}(0)$. The procedure to extract local eigenfunctions from the artificial ensembles is same as that in KL-representation. The stepwise methodology of eigenfunction extraction is as follows:

1. Fix the neighborhood size $\|\mathbf{B}(\underline{x}(0))\|$ and the length of the samples, i.e., the strand length L.

2. Start with the initial point $\underline{x}(0)$ on the trajectory.

3. Generate artificial ensembles as described in Section 2.

4. Extract local eigenfunctions from the artifical ensembles. The procedure is same as in KL representation.

5. Now shift to a new location on the trajectory and carry out steps (3) and (4).

12

The best set of local eigenfunctions may be obtained by choosing an appropriate combination of the neighborhood size $\|B(x(0))\|$ and the length of the samples L (strand length).

Using the extracted local eigenfunctions, we can develop a local eigenfunction representation. This is a piecewise signal representation based on locally extracted basis and is valid for almost all $x \in B(x(0))$. If we choose a sufficiently large neighborhood size, we may use the same set of locally extracted basis to represent longer intervals or more number of strands. Over a limit, if we shrink the ball to a singleton set (a set with only one element), the resulting representation becomes a time-domain representation itself. Thus, intuitively, the local eigenfunction representation is adequate to represent any signal in the L_2-space.

For chaotic signals, we may use the same set of basis functions, extracted using a sufficiently large-sized ball, to represent the whole signal. This claim is substantiated by the results of numerical experiments presented in the following:

3.1 Numerical performance evaluation

For the Rossler attractor, we set $d_E = 3$, initial location $\zeta(0) = [3.2012\,,\, 2.6967\,,\, 2.2156]^T$. We found $N_B = 40$ nearest neighbors in a neighborhood of $\zeta(0)$, as shown in Figure 6, and generated 40 ensembles therefrom. Next, we extracted local eigenfunctions from the generated ensembles. Only 6 eigenfunctions, shown in Figure 6, were found to be dominant and these alone were used for signal representation.

By appropriately shifting these 6 dominant eigenfunctions along the length of the signal using local exhaustive search, we were able to accurately represent the whole
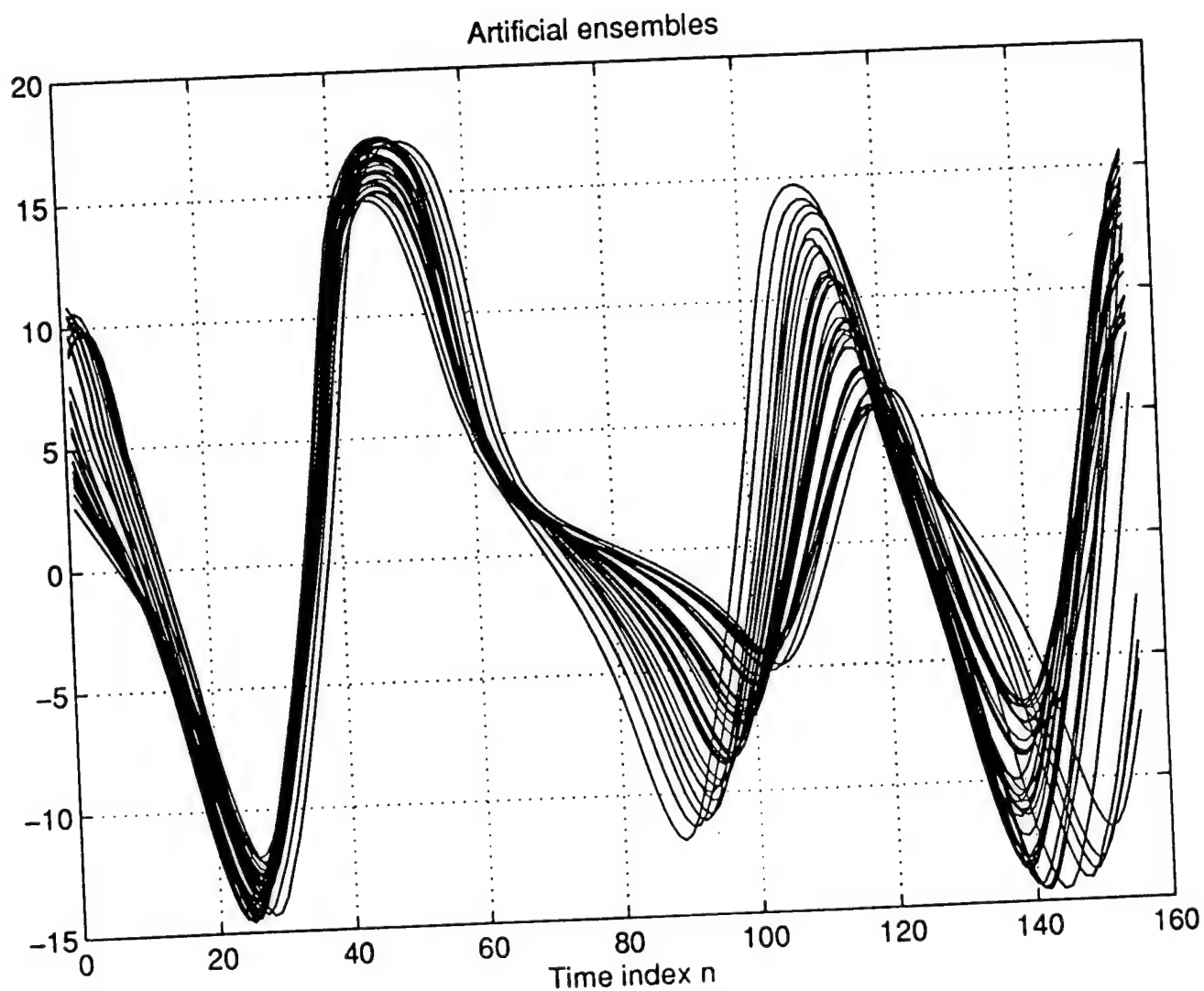
Figure 2: Artificial ensembles extracted from a single realization of a signal from the Rossler attractor. Here, $N_B = 40$, $d_E = 3$, lag $= 7$ and $L = 168$ (only 158 datapoint-long ensembles shown).
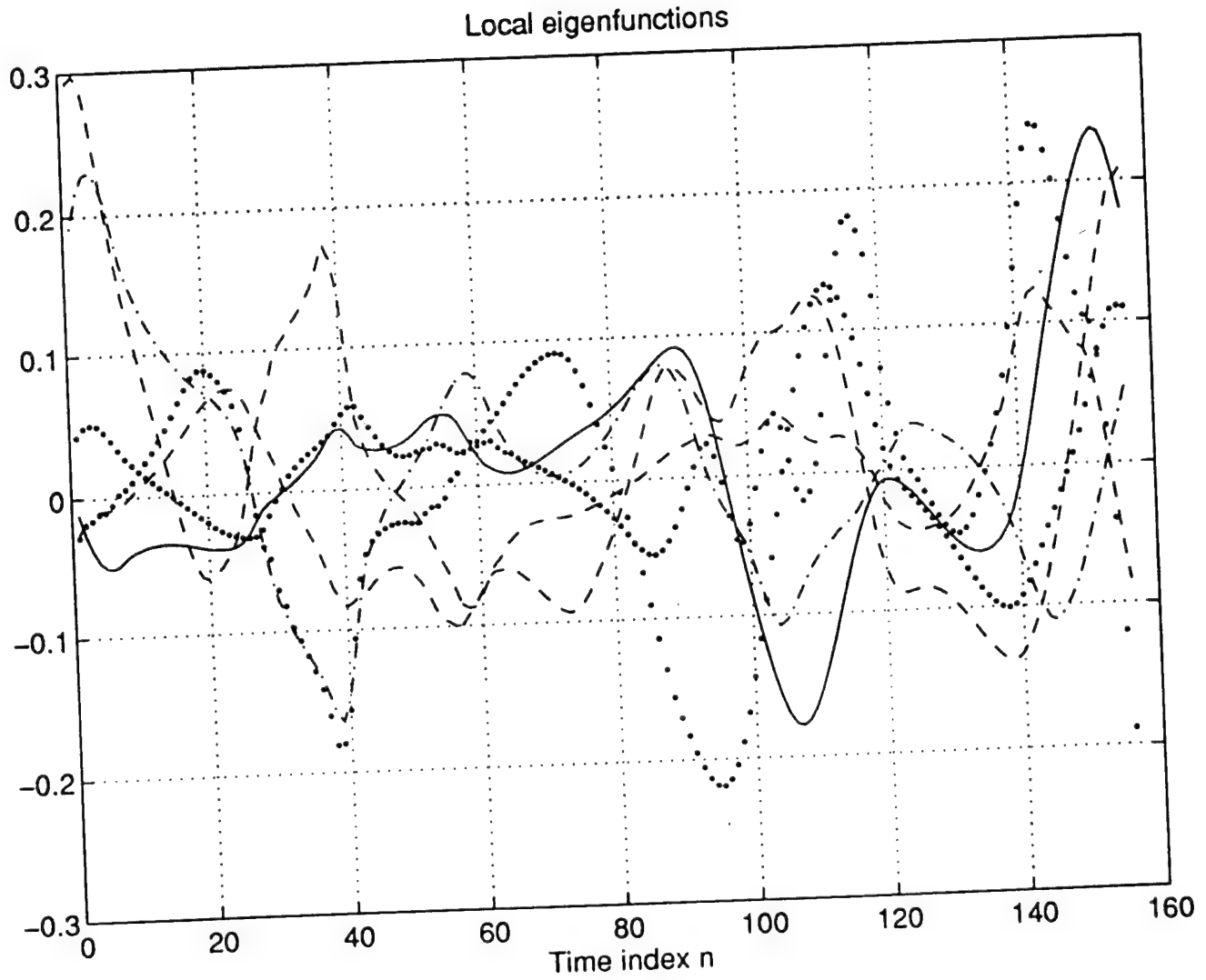
Figure 3: Six dominant local eigenfunctions for the Rossler attractor. Here, $N_B = 40$, $d_E = 3$, lag $= 7$ and $L = 168$ (only 158 datapoint-long eigenfunctions shown).

TSD within 8% of the total signal energy. This implies that the local eigenfunction representation is adequate for chaotic signals. The accuracy of the local eigenfunction is affected by the neighborhood size, the Lyapunov exponents of the TSD and the noise level because the considered signal emanates from an attractor, a compact invariant set. However, one major drawback of local eigenfunction representation is the computation time required to find appropriate shifts of eigenfunctions to represent the signal.

4 Suboptimal wavelet packet representation

We used a linear combination of dominant local eigenfunctions as the scaling function. Suppose α_j are the local eigenfunctions extracted from a chaotic signal using a sufficiently large Λ and

$$\bar{\lambda}_j = \frac{1}{N_B} \sum_{k=1}^{N_B} \bar{\lambda}_j^k,$$

(8)

where $\bar{\lambda}_j^k$ are the coefficients of $\alpha_j(t)$ while representing k^{th} ensemble. The scaling function is then given by

$$\phi(t) = \mathcal{S}(\sum_{j \in \mathbf{D}} \bar{\lambda}_j \, \alpha_j(t)),$$

(9)

where $\mathbf{D}$ is the set of dominant eigenfunctions. Here, $\mathcal{S}(\cdot)$ is the circular shift operator which makes the end points identically zero, and then appropriately decimates the resulting function. From the vector quantization standpoint, the undecimated function is a *codebook function* [3] in pseudo-probability space.

Table 1: Comparison of entropies resulting from wavelet packet representation of a signal from the Rossler attractor using different scaling functions.

Scaling function	Entropy
Coiflet 1	5.73
Daubechies 4	5.77
Daubechies 20	5.43
Cosine Packet	2.63
Suboptimal	0.03

We used the extracted scaling functions for WP representation of signals from the Rossler attractor. The length of the ensembles and hence the codebook function was set to $L = 168$. We decimated the codebook function by 4 to result in a scaling function 42 datapoints long. The computed scaling function is shown in Figure 6, and a comparison of *entropy*[3] resulting from WP representation with our scaling function against the entropies resulting from WP representation with other standard bases [12] is provided in Table 1. The entropy values of suboptimal representation are smaller than those of other standard bases by an order of magnitude. This implies that our suboptimal basis is more suited other standard bases for representing chaotic signals. A practical validation of our novel representation scheme is provided in the following section.

---

[3]Entropy is a measure of parsimony of representation. The smaller the value of signal entropy, the greater is the parsimony. [12]
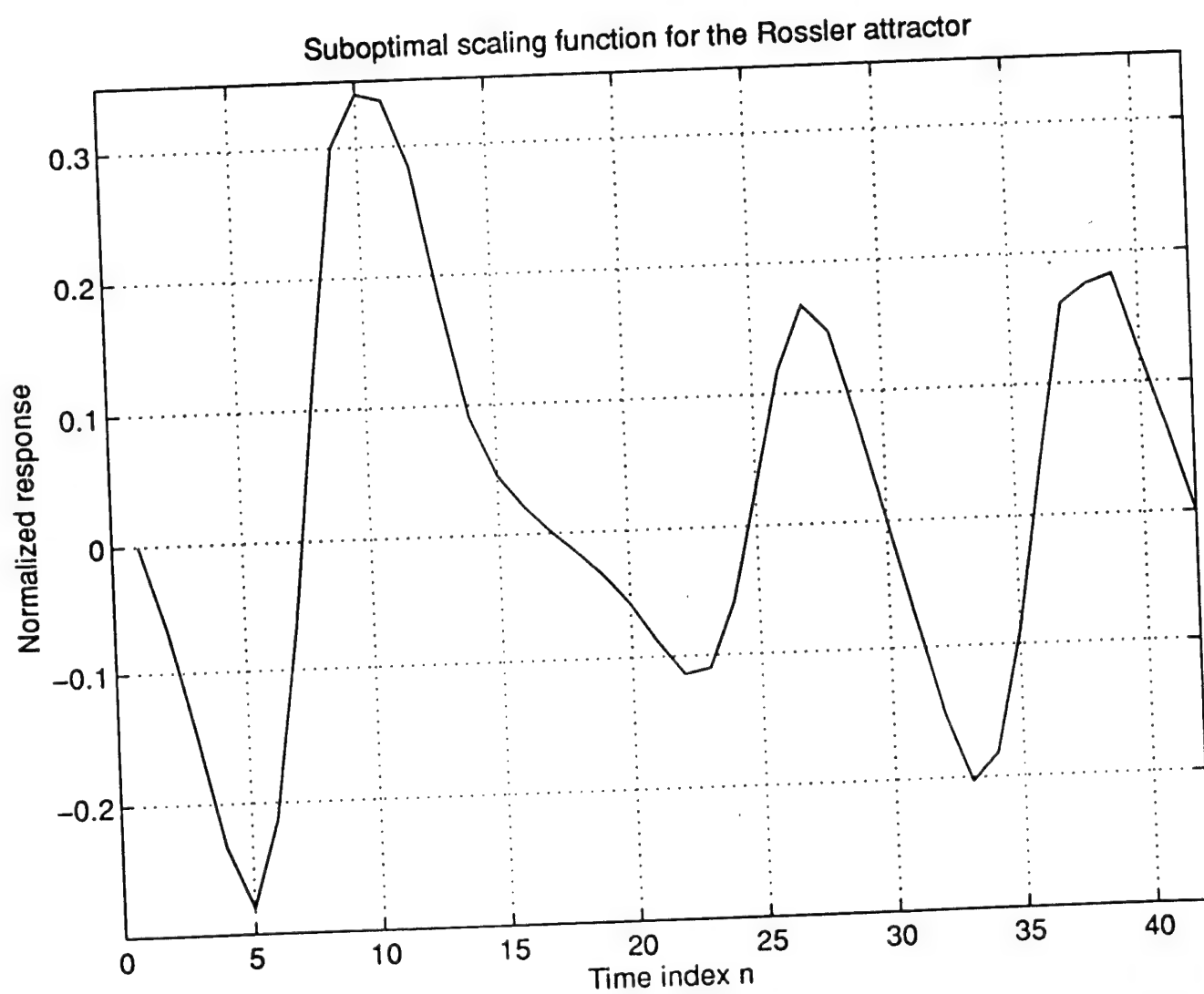
Figure 4: Scaling function (42 datapoints long) for suboptimal WP representation of the Rossler attractor.

Table 2: Comparison of entropies resulting from wavelet packet representation of a signal from the AE signals using different scaling functions.

Scaling function	Entropy
Coiflet 1	4.79
Daubechies 4	5.32
Daubechies 20	4.53
Cosine Packet	3.84
Suboptimal	0.05

5   Application to signals from machining sensors

We used the suboptimal wavelet basis to represent the acoustic emission (AE) signals in machining. For a partcular signal, with $L = 76$, $N_B = 50$ and $d_E = 9$, we computed 50 ensembles as shown in Figure 6, and determined the local eigenfunctions therefrom. Only 3 eigenfunctions, shown in Figure 6, were dominant. From these eigenfunctions, we constructed a scaling function as described earlier in this paper. The use of this scaling function substantially reduced the entropy of representation as revealed in Table 2. Clearly, the entropy of WP representation with the suboptimal scaling function is about a magnitude less than that for WP representation using other scaling functions, implying the practical effectiveness of our method.

6   Conclusion

We have thus developed a novel representation scheme for contaminated chaotic signals. It is probably for the first time that a representation scheme for contaminated chaotic
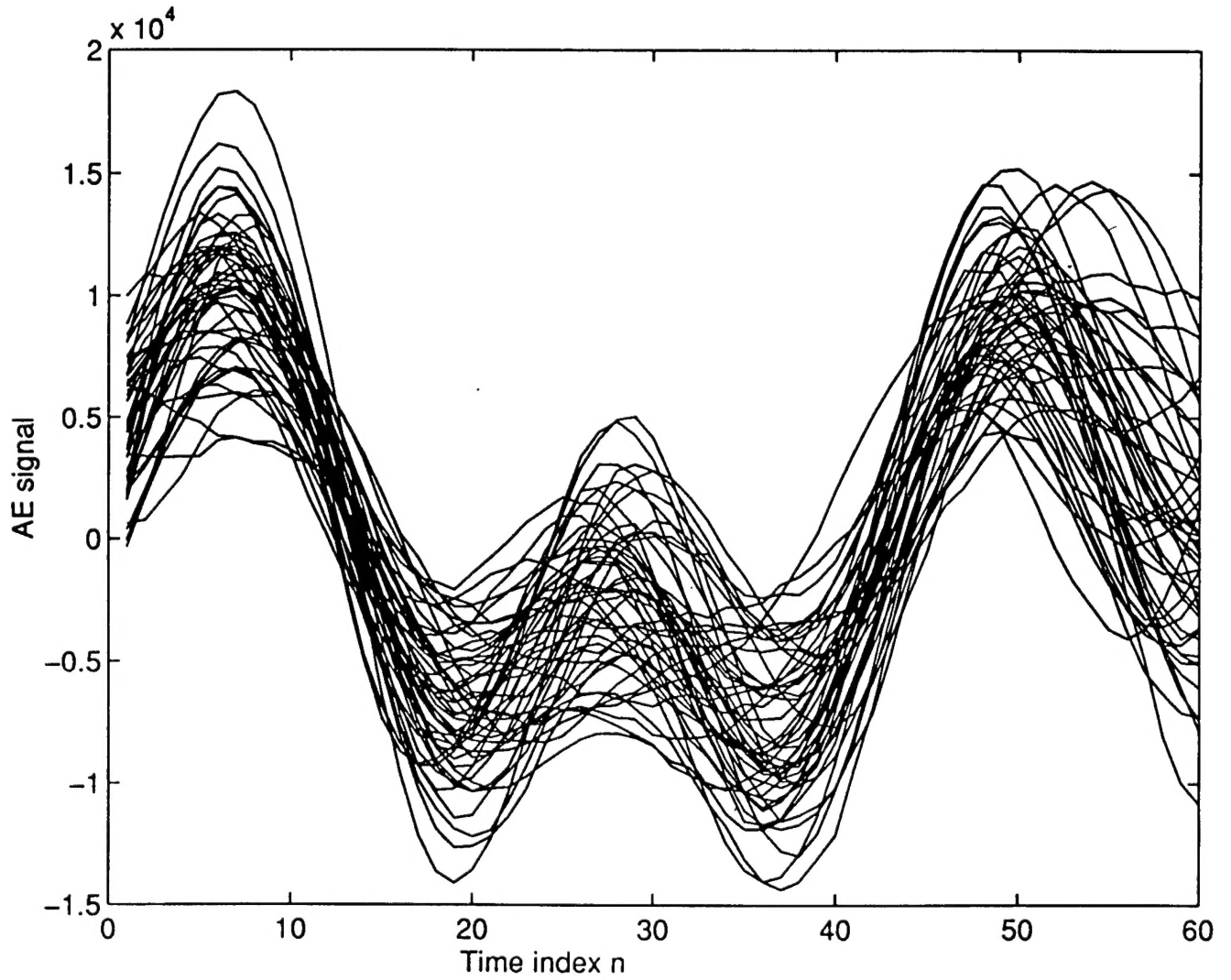
Figure 5: Artificial ensembles generated from an acoustic emission signal. The artificial ensembles were generated with $N_B = 50$, $d_E = 9$, lag $= 6$ and $\mathsf{L} = 60$.
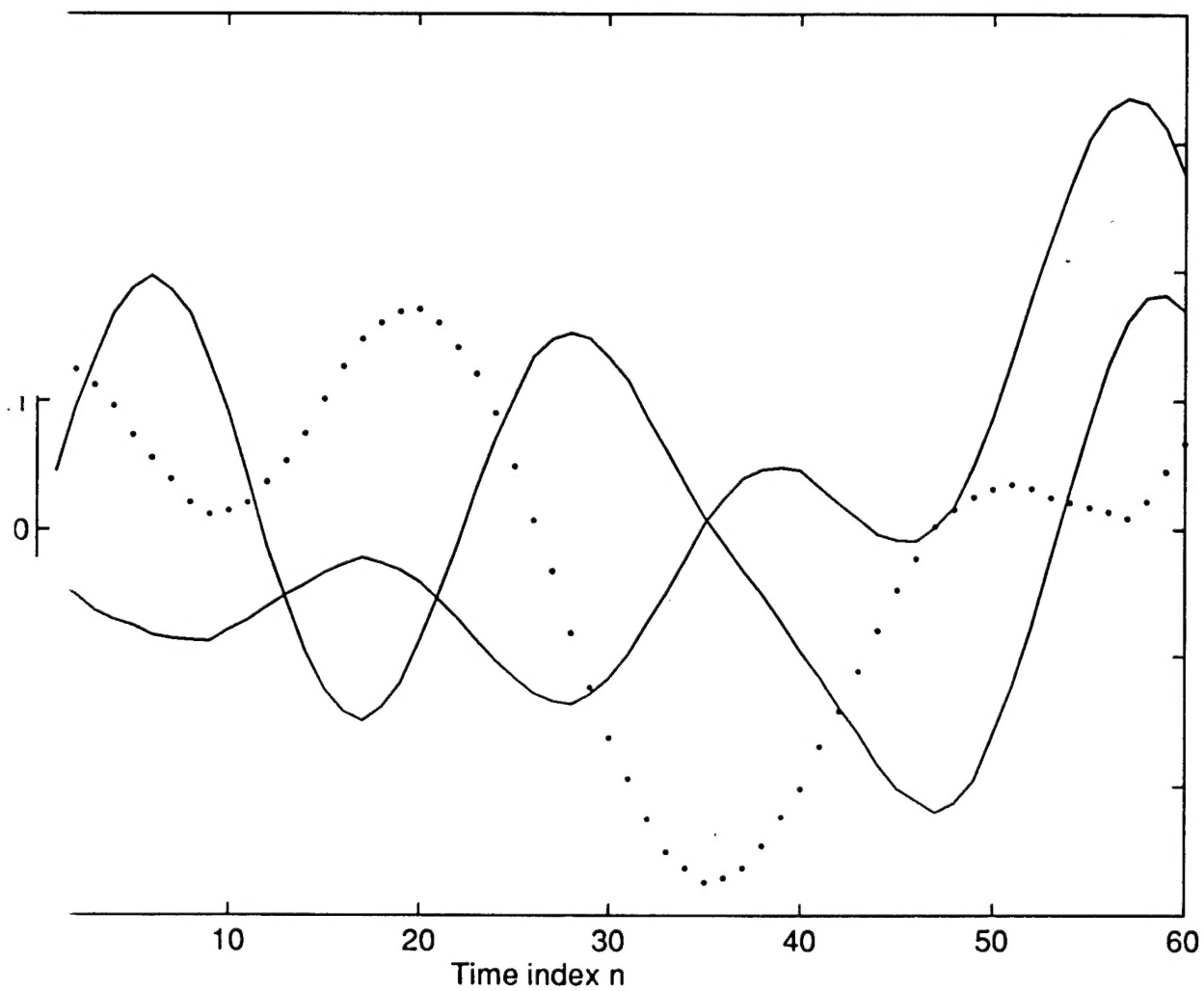
6: Three dominant eigenfunctions extracted from the artificial ensembles generated
an acoustic emission signal. The artificial ensembles were generated with $N_B = 50$,
), lag = 6 and L = 60.

signals has been developed. This parsimonious suboptimal representation scheme may be applied to various engineering disciplines where chaotic signals occur, and specifically to the machining process. In fact, we have met with reasonable success in using the scaling functions developed in this work to extract signal features from AE signals for tool wear estimation [2].

Acknowledgments

References

[1] Abarbanel, II. D. I., 1996. *Analysis of Observed Chaotic Data.* Springer–Verlag, New York.

[2] Bukkapatnam, S. T. S., 1997. Monitoring and control issues in chaotic processes: Application to turning. *Ph. D. Thesis*, Department of Industrial and Manufacturing Engineering, The Pennsylvania State University, University Park, 1997.

[3] Goldberg, A., 1993. Applications of wavelets to quantization and random process representations. *Ph. D. Thesis*, Department of Electrical Engineering, Stanford University, Stanford, CA.

[4] Isham, V., 1993. Statistical aspects of chaos: A review. In *Networks and Chaos— Statistical and Probabilistic Aspects*: Barndorff-Nielsen et al. (eds), Chapman and

Hall, London.

[5] Jazwinski, A., 1970. *Stochastic Processes and Filtering Theory.* Academic-Press, New York.

[6] Katok, A., & Hasselblatt, B., 1995. *Introduction to the Modern Theory of Dynamical Systems.* Cambridge, New York.

[7] Nayfeh, A. H., & Balachandran, B., 1995. *Applied Nonlinear Dynamics: Analytical, Computational and Experimental Methods.* Wiley, New York.

[8] Odegard, J. E., Gopinath, R. A., & Burrus, C. S., 1991. Optimal wavelets for signal decomposition and the existence of scale-limited signals. *Technical Report CML TR91-07*, Department of Electrical Engineering, Rice University, Houston, TX.

[9] Stark, H., & Wood, J., 1994. *Probability, Random Processes and Estimation Theory.* Prentice-Hall, Englewood Cliffs, NJ.

[10] Strintzis, M. G., 1996. Optimal biorthogonal wavelet bases for signal decomposition. *IEEE Transactions on Signal Processing*, 44, 1406–1417.

[11] Unser, M., 1993. On the optimality of ideal filters for pyramid and wavelet signal approximation. *IEEE Transactions on Signal Processing*, 41, 3591–3596.

[12] Wickerhauser, M. V., 1991. Lectures on wavelet packet algorithms. *Lecture Notes*, Department of Mathematics, Washington University, St. Louis, MO.

[13] Wiggins, S., 1990. *Introduction to Applied Nonlinear Dynamical Systems and Chaos.* Springer-Verlag, New York.